

# Defect-induced exceptional point in phonon lasing

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Exceptional points (EPs) are non-Hermitian degeneracies which occur when two or more modes of a physical system coalesce in their resonant frequency and their rate of decay. In systems where the loss of one mode is balanced by the gain in another mode, EPs are generally referred to as parity-time ( $\mathcal{PT}$ ) symmetry breaking point. EPs lead to many counter-intuitive phenomena such as loss-induced transparency, invisible sensing, loss-induced optical revival,  $\mathcal{PT}$  lasing or mode switching, and topological energy transfer. On the other hand, the emerging field of optomechanics has led to many important applications, including the striking observation of a phonon laser, which provides the core technology to integrate coherent phonon sources and devices. However, defects in materials, which have been traditionally considered to be detrimental for achieving phonon lasers, have been neglected in previous works. Here we show that contrary to this traditional view, material defects can lead to the emergence of an EP, beyond which the mechanical gain and the phonon number start to increase significantly with increasing loss. This indicates a novel and counterintuitive way to improve the performance of a phonon laser i.e., by tuning lossy defects, instead of adding any active dopants. In view of the controllability of defects via external electric fields, this also opens the way for electrically-tuned phonon lasing in optomechanical resonators.

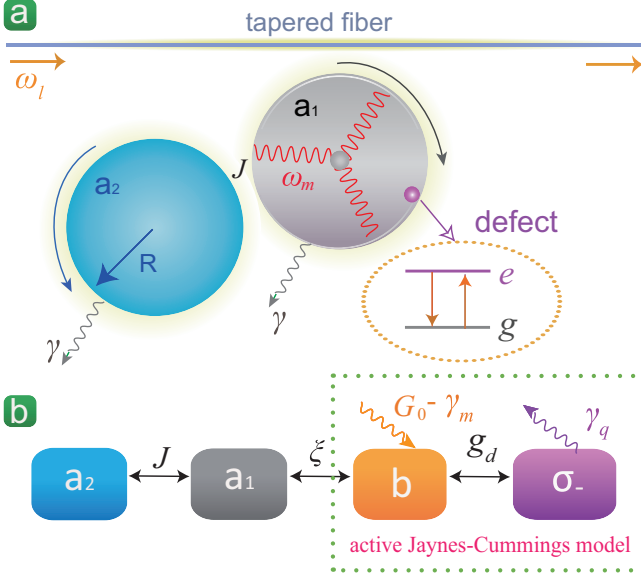
Rapid advances in cavity optomechanics (COM) [1, 2] in the past decade have led to novel applications, such as quantum ground-state cooling [3–5], single-photon transport [6], ultra-sensitive measurement [7–9], and phonon lasing [10–16]. As an acoustic analog of photon lasers, coherent stimulated amplification of sound (phonon laser or phaser) was demonstrated in various COM systems [13, 17, 18]. Mode competition [17] and sub-Poissonian distribution [18] have been observed for phonons, opening up the way to study nonlinear phononics and to build functional phonon devices. The first COM experiment on phonon lasing was based on two coupled optical microresonators, one of which supports a mechanical mode [13]. This compound system has two optical supermodes, with the frequency difference  $\Delta\omega$  determined by the inter-cavity coupling strength [19, 20]. When  $\Delta\omega$  is equal to the frequency of the mechanical mode, the COM coupling enables energy exchange between the optical supermodes through phonon-mediated transitions [13], leading to low-threshold phonon amplification and lasing [13, 21, 22]. This is analogous to a photon laser, where transitions between two electronic levels of an atom is mediated by photon absorption and emission processes. It was demonstrated theoretically [22, 23] that if the optical loss of the resonator supporting the mechanical mode is balanced with gain provided by the other resonator, optomechanical interactions and gain can be enhanced

to provide thresholdless  $\mathcal{PT}$ -symmetric phonon lasing. More recently, we showed that [24] the same system can give rise to a high-order EP (i.e., with three coalescing eigenfrequencies and eigenvectors) that significantly increases phonon cooling rates. The required optical gain in such  $\mathcal{PT}$ -symmetric coupled COM systems is provided either by doping optically-active materials (e.g., rare-earth ions, dyes, etc) into the resonator or by using the intrinsic gain mechanisms of the material the resonator is made from, such as Raman and parametric gain.

All experimental and theoretical analysis on phonon lasers have so far neglected the impact of two-level-systems (TLSs) emerged due to defects, which naturally exist in bulk amorphous materials and mechanical resonators. Here we propose to use intrinsic TLS defect states for steering a coupled optomechanical system to one of its EPs to control optomechanical interactions and for obtaining  $\mathcal{PT}$ -symmetric phonon lasing without introducing dopants. TLS defects were first studied within the context of thermal and acoustic properties of amorphous glasses at cryogenic temperatures [25–27]. They are ubiquitous in amorphous materials which are used in the fabrication of nanoelectromechanical and optomechanical systems, and quantum devices. TLSs can couple to different modes of a system via different mechanisms, e.g., to superconducting qubits via electric dipole moment and to a mechanical mode via strain or deformation force. For many years, they were considered as the source of loss and decoherence, and as such, techniques have been developed to decrease the number of TLS, if not to remove them completely, from the material sys-

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**Figure 1: Defect-assisted phonon lasing.** (a) The COM system for phonon lasing is composed of two lossy optical microresonators coupled to each other with an inter-resonator tunneling rate  $J$ . These microresonators have the same optical decay rate  $\gamma$ . The resonator coupled to a tapered optical fiber and driven by a pump field with frequency  $\omega_l$  supports a mechanical mode at frequency  $\omega_m$  [13] and contains a defect-induced TLS. The intracavity optical fields of the resonators are denoted by  $a_1$  and  $a_2$ , the phonon annihilation operator is  $b$ , and  $R$  is the radius of the resonators. (b) The interaction model for the defect-assisted phonon lasing: the optical mode  $a_1$ , which is coupled to the mode  $a_2$  with strength  $J$ , interacts with the mechanical mode  $b$ , which, in turn, is coupled to the defect TLS, represented by the Pauli operator  $\sigma_-$ , by a coupling strength  $g_d$ . The decay rates of the phonon mode and the defect are denoted by  $\gamma_m$  and  $\gamma_q$ , respectively. The term  $\gamma'_m = \gamma_m - G_0$  is the effective mechanical damping rate, where  $G_0$  corresponds to mechanical gain.

tem of interest [28]. However, recent studies have shown that TLS can play useful roles. For example, they have been used as quantum memory in superconducting circuits [29, 30] and as random-defect lasers [31]. In COM devices, the presence of TLS defects can affect COM interactions [32], mechanical resonance and damping [9], ground-state cooling [33–35], and nonlinear responses of COM devices [36]. We note that all previous studies on phonon lasers have so far neglected the role of defect naturally occurring in the materials used to fabricate COM resonators.

Here, we explore the impact of intrinsic defect-induced TLS on phonon lasing in compound microresonators. We demonstrate theoretically that when the damping rate of the TLS defect is below a critical value, the mechanical gain of the system is suppressed with increasing damping. Beyond this critical damping rate, however, increasing the damping rate enhances the mechanical gain. We show that this counterintuitive phenomenon is the result of the emergence of an exceptional point (EP) in the

phonon lasing regime due to coupling between the lossy TLS mode and the active mechanical mode, which creates an effective parity-time ( $\mathcal{PT}$ )-symmetric system. This is strongly reminiscent of the situation encountered in loss-induced suppression and revival of optical lasing in the vicinity of an EP [37]. The unconventional role of the EP here is identified as the defect-induced enhancement of phonon lasing, including the enhanced mechanical gain and the lowered threshold power. Our results indicate that the performance of phonon lasers can be enhanced and controlled by manipulating lossy defects and hence the TLS defects in the resonator supporting the mechanical mode. This opens the possibility of tuning phonon lasers electrically due to the fact that TLS defects can be coherently driven by microwave fields. We note here that both of the resonators forming the COM device in this study are lossy optical resonators. This is different from the COM device used for the  $\mathcal{PT}$ -symmetric phonon laser in Ref. [22], where one of the resonators had optical gain provided by dopants to compensate the loss of the other resonator.

## Results

**System.** The system we consider in this study is depicted in Fig. 1. Two whispering-gallery-mode (WGM) microresonators having the same resonance frequency  $\omega_c$  and the loss rate  $\gamma$  are coupled to each other with a coupling strength of  $J$  which can be tuned by varying the distance between the resonators. One of the resonators supports a mechanical mode with effective mass  $m$ , frequency  $\omega_m$  and damping rate  $\gamma_m$ . We consider resonators made of silica, silicon, or silicon nitride, where intrinsic TLSs exist. Also, the resonator supporting the mechanical mode has a defect that is coupled to the local phonon mode via mechanical strain, with the coupling strength [34]:

$$g_d \approx \frac{D_T}{\hbar} \frac{\Delta_0}{\omega_q} \sqrt{\frac{\hbar \omega_m}{2YV_m}}, \quad (1)$$

where  $\omega_q$  is the tunable energy difference between the excited and ground states of the defect-induced TLS [27],  $D_T$  denotes the potential of mechanical deformation,  $Y$  is the Young's modulus of the material, and  $V_m$  is the mechanical mode volume [36].

In the rotating frame at the pump frequency  $\omega_l$ , the Hamiltonian of this system, which is composed of two optical modes, one mechanical mode and one TLS, can be written as

$$\begin{aligned} H &= H_0 + H_{\text{int}} + H_{\text{dr}}, \\ H_0 &= -\Delta(a_1^\dagger a_1 + a_2^\dagger a_2) + \omega_m b^\dagger b + \frac{\omega_q}{2} \sigma_z, \\ H_{\text{int}} &= J(a_1^\dagger a_2 + a_2^\dagger a_1) - \xi a_1^\dagger a_1 x + g_d(b^\dagger \sigma_- + \sigma_+ b), \\ H_{\text{dr}} &= i(\varepsilon_l a_1^\dagger - \varepsilon_l^* a_1), \end{aligned} \quad (2)$$

where  $a_1$ ,  $a_2$  or  $b$  denote the annihilation operators of the optical modes or the mechanical mode,  $x = x_0(b^\dagger + b)$  is the mechanical displacement operator,  $\Delta \equiv \omega_l - \omega_c$  denotes the frequency detuning between the pump laser and the cavity resonance,  $\xi = \omega_c/R$  is the COM coupling strength,  $R$  is the resonator radius,  $x_0 = (1/2m\omega_m)^{1/2}$ , and  $\sigma_z$ ,  $\sigma_-$  and  $\sigma_+$  are the Pauli operators of the TLS (see Fig. 1) given by  $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ ,  $\sigma_- = |g\rangle\langle e|$ , and  $\sigma_+ = |e\rangle\langle g|$ . The field amplitude of the pump light is given by  $\varepsilon_l = (2P_l\gamma/\hbar\omega_l)^{1/2}$ , where  $P_l$  is the power of the pump.

Defining the supermode operators for the optical fields as  $a_\pm = (a_1 \pm a_2)/\sqrt{2}$  and applying the rotating-wave approximation, we obtain the effective interaction Hamiltonian of the system:

$$\mathcal{H}_{\text{int}} = -\frac{\xi x_0}{2}(a_+^\dagger a_- b + b^\dagger a_+ a_-^\dagger) + g_d(b^\dagger \sigma_- + \sigma_+ b). \quad (3)$$

The first term in this expression describes the phonon-mediated transition between optical supermodes, and the second term describes the coupling between the phonon and TLS defect. Thus, in the supermode picture, the optomechanical coupling is transformed into an effective coupling describing defect-assisted phonon lasing. The TLS can be excited by absorbing a phonon generated from the transition between the upper optical supermode and the lower one, so that the behaviour of the phonon lasing is strongly modified.

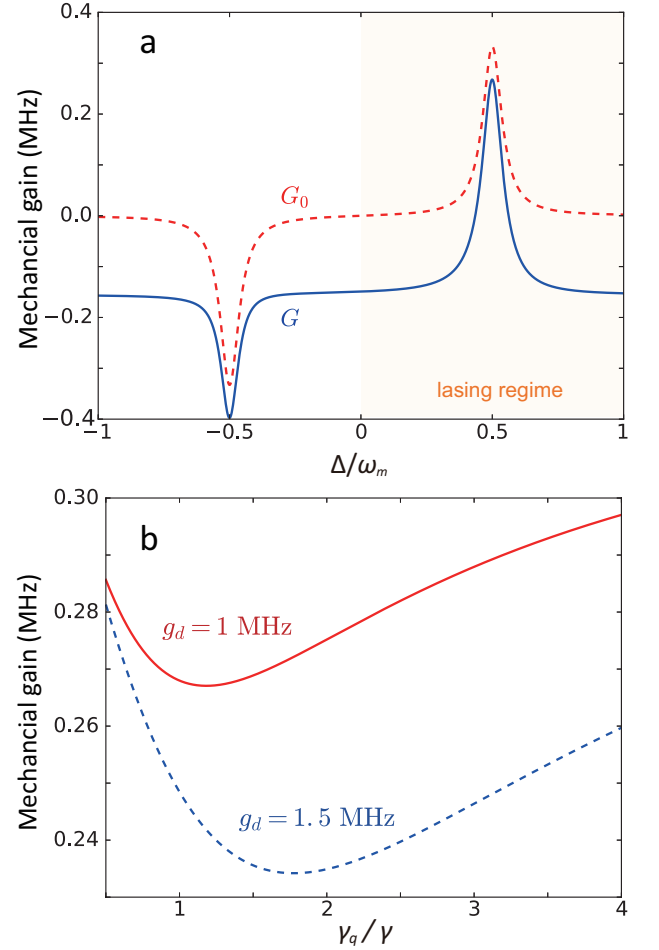
By setting the time derivative of the operators  $a_+$ ,  $a_-$ ,  $\sigma_-$ , and  $p = a_-^\dagger a_+$  to zero, we can obtain the mechanical gain. As described in the Methods and the Supplement, the mechanical gain of this system is given by

$$\begin{aligned} G &= G_0 + G_d, \\ G_0 &= \frac{\xi^2 x_0^2 \gamma}{2(2J - \omega_m)^2 + 8\gamma^2} \left[ \delta n - \frac{\Delta(2J - \omega_m)|\varepsilon_l|^2}{\alpha^2 + 4\Delta^2\gamma^2} \right], \\ G_d &= -\frac{g_d^2 \gamma_q}{\gamma_q^2 + (\omega_q - \omega_m)^2 + 2g_d^2 n_b}, \end{aligned} \quad (4)$$

where  $\gamma_q$  is the defect TLS decay rate, and  $\delta n = a_+^\dagger a_+ - a_-^\dagger a_-$  is the population inversion operator, and

$$\alpha = J^2 + \gamma^2 - \Delta^2 + \frac{\xi^2 x_0^2}{4} n_b,$$

with the phonon number  $n_b = b^\dagger b$  (see the supplementary materials). If there is no defect, i.e.  $g_d = 0$ , we have  $G = G_0$ , as was already obtained in Ref. [21]. The second term of  $G_0$ , describing the optical detuning effect, can be made positive or negative by tuning the optical tunneling rate  $J$  (i.e., coupling strength of the resonators) by changing the distance between the resonators. Note that this term is positive when  $2J - \omega_m > 0$ , negative when  $2J - \omega_m < 0$ , and zero when  $\delta = 0$  or  $2J = \omega_m$ . In the latter case,  $G_0$  is described with only the first term, i.e.  $G_0 = \xi^2 x_0^2 / 8\gamma$ , as given in Ref. [13]. The inevitable effect of material defects on the phonon lasing, as described by  $G_d$ , has not been studied before.



**Figure 2: Mechanical gain in the defect-assisted phonon laser.** (a) The mechanical gains  $G_0$  and  $G$  as a function of the optical detuning  $\Delta$ . (b) The mechanical gain  $G$  as a function of the TLS decay rate  $\gamma_q$ . We choose  $\gamma_q = \gamma$  and  $g_d = 1$  MHz in (a). The inter-resonator coupling rate is set as  $J = 0.5\omega_m$  in (a,b) and the optical detuning is set as  $\Delta = 0.5\omega_m$  in (b). Other tunable parameters are chosen as  $\omega_q = \omega_m$  and  $P_l = 10 \mu\text{W}$ .

With the mechanical gain at hand, we can derive the threshold pump power  $P_{\text{th}}$  from the threshold condition  $G = \gamma_m$  (see the supplementary materials). By setting  $J = \omega_m/2$  and  $\omega_q = \omega_m$ , the threshold power  $P_{\text{th}}$  can be simplified to

$$P_{\text{th}} \approx \frac{8\gamma^2(\omega_c + J)}{(\xi x_0)^2} \left( \gamma_m + \frac{g_d^2 \gamma_q}{\gamma_q^2 + 2g_d^2 n_b} \right). \quad (5)$$

**Mechanical gain.** Figure 2(a) shows the mechanical gain  $G_0$  and  $G$  as a function of the optical detuning  $\Delta$  with  $J = 0.5\omega_m$ , using the experimentally feasible parameter values [13, 20], i.e.  $R = 34.5 \mu\text{m}$ ,  $m = 50 \text{ ng}$ ,  $\omega_c = 193 \text{ THz}$ ,  $\omega_m = 2\pi \times 23.4 \text{ MHz}$ ,  $\gamma = 6.43 \text{ MHz}$  and  $\gamma_m = 0.24 \text{ MHz}$ . Clearly, in the cooling regime (with  $\Delta < 0$ ),  $G$  is negative and can be further enhanced by the defects (as firstly revealed in Ref. [36]). The positive

gain in the lasing regime (with  $\Delta > 0$ ) is also significantly affected by the TLS-phonon coupling. In particular, the phonon lasing is most preferable at  $\Delta/\omega_m \sim 0.5$ , where the defect-induced reduction in  $G$  is minimized. If we further set  $J = 0.5\omega_m$ , optimized phonon lasing is achieved (see the supplementary materials). More interestingly, Fig. 2(b) shows an unexpected evolution for  $G$  with increasing defect loss rate: the mechanical gain  $G$  first decreases with increasing TLS decay rate, until a critical value of  $\gamma_q$ . When this exceeds the critical value,  $G$  increases with increasing defect loss. Consequently the phonon laser threshold power  $P_{th}$  first increases and then decreases again with increasing TLS damping, as shown in the supplementary materials. These counterintuitive results are reminiscent of the loss-induced suppression and revival of lasing [37, 38] and the loss-induced transparency [39] demonstrated previously in coupled optical resonators and coupled waveguides.

**Supermode spectrum.** To understand this unconventional loss-induced effect intuitively, we use a simplified model to describe the defect-assisted lasing process, with only the *active* phonon mode and the lossy TLS. The effective Hamiltonian of this simplified model reads

$$\mathcal{H}_{\text{eff}} = (\omega_m - i\gamma'_m)b^\dagger b + (\omega_q - i\gamma_q)\sigma_+\sigma_- + g_d(b^\dagger\sigma_+ + \sigma_-b), \quad (6)$$

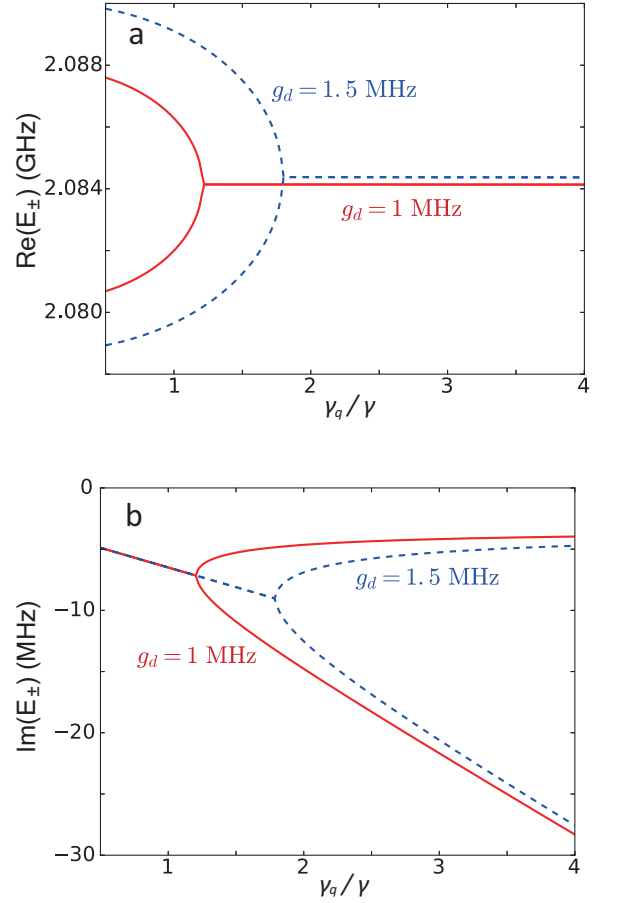
with the effective mechanical damping rate

$$\gamma'_m = \gamma_m - G_0. \quad (7)$$

We note that a similar route was adopted in a recent experiment for an (anti-) $\mathcal{PT}$ -symmetric atomic system [40] where, by starting from a Hermitian Hamiltonian describing atom-light interactions, an effective non-Hermitian Hamiltonian was deduced for two spin-wave excitation channels (see also Ref. [41]). Choosing two basis states  $|n_b, g\rangle$  and  $|n_b - 1, e\rangle$  to diagonalize  $\mathcal{H}_{\text{eff}}$  leads to the eigenvalues

$$E_{\pm} = \left(n_b - \frac{1}{2}\right)\omega_m + \frac{\omega_q}{2} - \frac{i}{2}[(2n_b - 1)\gamma'_m + \gamma_q] \pm \frac{1}{2}\sqrt{4n_bg_d^2 + [\omega_q - \omega_m - i(\gamma_q - \gamma'_m)]^2} \quad (8)$$

The supermode spectrum (real and imaginary parts of these eigenvalues) is shown in Fig. 3(a,b), where an EP is clearly seen, at the positions close to the turning points in Fig. 2(b). The emergence of an EP in this system can be explained as follows: The composite system corresponds to a  $\mathcal{PT}$ -symmetric system where the mechanical gain in the optomechanical resonator partially or completely compensates the loss of the defect TLS. When  $\gamma_q$  is smaller than a critical value, the system is in the unbroken (exact)  $\mathcal{PT}$ -symmetric phase where the supermodes are almost equally distributed between the optomechanical resonator and the defect TLS, and the active phonon mode partially or completely compensates the loss induced by the defect TLS. As a result, the system has less net mechanical gain as the defect loss  $\gamma_q$  is increased. If

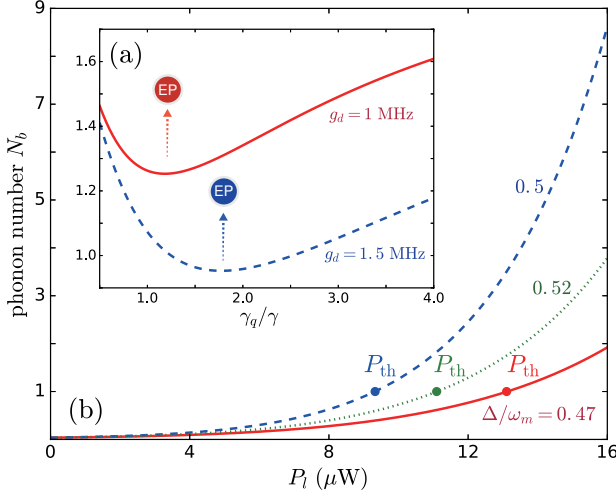


**Figure 3: Super-mode spectrum.** Real (a) and imaginary (b) parts of the eigenvalues of  $\mathcal{H}_{\text{eff}}$ . The other parameters are chosen as  $J = 0.5\omega_m$ ,  $\gamma_q = \gamma$ , and  $P_l = 7 \mu\text{W}$ .

$\gamma_q$  increases beyond the critical value, the supermodes become increasingly localized such that one dominantly resides in the optomechanical resonator and the other in the TLS. This is the broken- $\mathcal{PT}$  phase. As a result, the supermode in the optomechanical resonator experiences less and less loss with increasing  $\gamma_q$ , leading to an increasing net mechanical gain. The supermode which is dominant in the TLS, on the other hand, experiences increasingly more loss and has negligible effect on the mechanical gain. The transition from the exact- to broken- $\mathcal{PT}$  phase takes place at an EP [42–44], and we label the critical value of  $\gamma_q$  at the EP as  $\gamma_q^{\text{EP}}$ . Thus in this composite system of the optomechanical resonator and defect TLS, the region  $\gamma_q < \gamma_q^{\text{EP}}$  corresponds to the exact  $\mathcal{PT}$ -phase, whereas the region where  $\gamma_q > \gamma_q^{\text{EP}}$  corresponds to the broken- $\mathcal{PT}$  phase. Note that at  $\omega_q = \omega_m$ , the EP of the system emerges at  $\gamma_q^{\text{EP}} = \gamma'_m + 2g_d\sqrt{n_b}$ , while the turning point of  $G$  is obtained, by setting  $\partial G/\partial\gamma_q = 0$ , as

$$\gamma_q^{\text{min}} = \sqrt{2n_b} g_d. \quad (9)$$

The slight shift of the turning point from the exact EP position is due to the fact that  $\gamma_q^{\text{min}}$  depends on  $\Delta$ , while



**Figure 4: Stimulated emitted phonon number.** The phonon number  $N_b$  versus the pump power  $P_l$  (a) and the damping rate  $\gamma_q$  (b), with the fixed value  $\omega_q/\omega_m = 1$ . Also we take  $P_l = 10 \mu\text{W}$  in (a), and  $\gamma_q/\gamma = 1$ ,  $g_d = 1$  MHz in (b).

the EP does not (see also Ref. [37] and the supplementary materials). We note that the mechanism outlined here for our composite system formed by the COM resonator and defect is similar to that which led to an EP observed in an open quantum system composed of a single atom and a high- $Q$  optical cavity [45].

**Phonon number in the lasing region.** Figure 4 shows the dependence of the phonon number

$$N_b = \exp[2(G - \gamma_m)/\gamma_m]$$

on the defect loss and the pump power. Features similar to those observed for the mechanical gain also appear for  $N_b$ , i.e. more loss leads to the suppression of  $N_b$  in the  $\mathcal{PT}$ -symmetric regime but it is enhanced in the  $\mathcal{PT}$ -breaking regime. The tuning point of  $N_b$  is in exact correspondence with that of the mechanical gain, as shown in Fig. 2(b) or the threshold power (see the supplementary material). Figure 4(b) shows that  $N_b$  is strongly dependent on the optical detuning, and the optimized condition  $\Delta = 0.5\omega_m$ , as in the situation without any defect, still holds in this system.

### Discussion

In summary, we have studied the defect-assisted phonon lasing in coupled passive resonators. We find that, in contrast to intuitive expectations, the mechanical gain is not always suppressed with more defect loss. The exact evolutions of the mechanical gain and the threshold power, starting from the Hermitian full system, depicts a turning point as the loss is increased. This is found to be closely related to the emergence of an EP in an effective non-Hermitian phonon-defect system. By suitably choosing the system parameters, the active phonon mode and

the lossy defect mode, coupled via the mechanical strain [36], can be enforced into an unconventional  $\mathcal{PT}$ -breaking regime, where both the mechanical gain and the phonon number are enhanced despite increasing loss.

Our findings suggest an acoustic analog of loss-induced suppression and revival of an optical laser demonstrated in experiments [37, 38]. This indicates a novel and counterintuitive way to improve the performance of a phonon laser; that is by tuning the lossy defects, instead of adding any active dopants. This is important because the TLS energy splitting and damping rate are not only determined by the material strain [36, 46], but they can also be controlled by steering external electric fields [31], opening the way for electrically-tuned phonon lasing in COM systems. Our future works along this direction includes the study of EP-assisted quantum nonlinear COM [47, 48], the defect-induced transparency [2], and the defect-mediated sensing or transducing [49, 50]. Finally, we note that although we have developed the formalism for COM systems, the same concept can be extended to nanoelectromechanical devices in which phonon lasing has been achieved [15].

### Methods

**Optomechanics in the supermode picture.** To derive the Hamiltonian in the supermode picture, we define the supermode operator  $a_{\pm} = (a_1 \pm a_2)/\sqrt{2}$ , which transforms  $H_0$  and  $H_{\text{dr}}$  into

$$\begin{aligned} \mathcal{H}_0 &= \omega_+ a_+^\dagger a_+ + \omega_- a_-^\dagger a_- + \omega_m b^\dagger b + \frac{\omega_q}{2} \sigma_z, \\ \mathcal{H}_{\text{dr}} &= \frac{i}{\sqrt{2}} \left[ \varepsilon_l (a_+^\dagger + a_-^\dagger) - \text{h.c.} \right], \end{aligned} \quad (10)$$

with  $\omega_{\pm} = -\Delta \pm J$ . Similarly,  $H_{\text{int}}$  becomes

$$\begin{aligned} \mathcal{H}_{\text{int}} &= -\frac{\xi x_0}{2} \left[ (a_+^\dagger a_+ + a_-^\dagger a_-) - (a_+^\dagger a_- + \text{H.c.})(b^\dagger + b) \right] \\ &\quad + g_d (b^\dagger \sigma_- + \sigma_+ b). \end{aligned} \quad (11)$$

In the rotating frame with respect to  $\mathcal{H}_0$ , we have

$$\begin{aligned} \mathcal{H}_{\text{int}} &= -\frac{\xi x_0}{2} \left( a_+^\dagger a_- b e^{i(2J - \omega_m)t} + \text{H.c.} \right) \\ &\quad - \frac{\xi x_0}{2} \left( a_+^\dagger a_- b^\dagger e^{i(2J + \omega_m)t} + \text{H.c.} \right) \\ &\quad + \frac{\xi x_0}{2} \left( a_+^\dagger a_+ + a_-^\dagger a_- \right) (b^\dagger e^{i\omega_m t} + \text{H.c.}) \\ &\quad + g_d \left[ b^\dagger \sigma_- e^{i(\omega_m - \omega_q)t} + \text{H.c.} \right]. \end{aligned} \quad (12)$$

Considering the rotating-wave approximation

$$2J + \omega_m, \omega_m \gg |2J - \omega_m|, |\omega_q - \omega_m|,$$

we have

$$\mathcal{H}_{\text{int}} = -\frac{\xi x_0}{2} (a_+^\dagger a_- b + b^\dagger a_+ a_-^\dagger) + g_d (b^\dagger \sigma_- + \sigma_+ b). \quad (13)$$

The Heisenberg-Langevin equations of motion of this system are given by

$$\begin{aligned}
\dot{a}_+ &= (-i\omega_+ - \gamma)a_+ + \frac{i\xi x_0}{2}a_-b + \frac{\varepsilon_l}{\sqrt{2}} + \sqrt{\gamma}a_{\text{in}}, \\
\dot{a}_- &= (-i\omega_- - \gamma)a_- + \frac{i\xi x_0}{2}a_+b^\dagger + \frac{\varepsilon_l}{\sqrt{2}} + \sqrt{\gamma}a_{\text{in}}, \\
\dot{b} &= (-i\omega_m - \gamma_m)b + \frac{i\xi x_0}{2}a_+^\dagger a_- - ig_d\sigma_- + \sqrt{2\gamma_m}b_{\text{in}}, \\
\dot{\sigma}_- &= (-i\omega_q - \gamma_q)\sigma_- + ig_db\sigma_z + \sqrt{2\gamma_q}\Gamma_-, \\
\dot{\sigma}_z &= -2\gamma_q(\sigma_z + 1) - 2ig_d(\sigma_+b - b^\dagger\sigma_-) + \sqrt{2\gamma_q}\Gamma_z.
\end{aligned} \tag{14}$$

Here  $a_{\text{in}}$ ,  $b_{\text{in}}$ ,  $\Gamma_-$ , and  $\Gamma_z$  denote environmental noises corresponding to the operators  $a$ ,  $b$ ,  $\sigma_-$  and  $\sigma_z$ . We assume that the mean values of these noise operators are zero, i.e.

$$\langle a_{\text{in}} \rangle = \langle b_{\text{in}} \rangle = \langle \Gamma_- \rangle = \langle \Gamma_z \rangle = 0.$$

The fluctuations are small and we neglect the noise operators in our numerical calculations. Then the defect-assisted mechanical gain and the threshold power of the phonon lasing can be obtained, see the main text (for more details of the derivations and more related results, see the Supplementary Materials).

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### Acknowledgement

We thank Y. Jiang and Y.-Z. Wang for discussions. H. J. is supported by the National Natural Science Foundation of China under Grand numbers 11274098 and 11474087. S. K. Ö. is supported by ARO Grant No. W911NF-16-1-0339. F. N. is supported by the RIKEN iTHES Project, the MURI Center for Dynamic Magnetooptics via the AFOSR award number FA9550-14-1-0040, the IMPACT program of JST, CREST, and a Grant-in-Aid for Scientific Research (A).

### Author contributions statement

H. J. conceived the idea and performed the calculations with the help from H. L.; H. J. and S. K. Ö. wrote the manuscript with input from F. N. and L. M. K.; all the authors discussed the content of the manuscript.

### Additional information

Competing financial interests: The authors declare no competing financial interests.

## Supplementary Materials for “Defect-induced exceptional point in phonon lasing”

### I. DERIVATION OF THE MECHANICAL GAIN

In the super-mode picture, a crucial term describing the phonon-lasing process can be resonantly chosen from the Hamiltonian, under the rotating-wave approximation [S1] (for  $J \sim \omega_m/2$ ,  $\omega_q \sim \omega_m$ , see the Methods). With the supermode operators  $p = a_-^\dagger a_+$ ,  $a_\pm = (a_1 \pm a_2)/\sqrt{2}$ , the reduced interaction Hamiltonian is given by

$$\mathcal{H}_{\text{int}} = -\frac{\xi x_0}{2}(p^\dagger b + b^\dagger p) + g_d(b^\dagger \sigma_- + \sigma_+ b). \quad (\text{S1})$$

The resulting Heisenberg equations of motion then read as [S1]

$$\begin{aligned} \dot{p} &= (-2iJ - 2\gamma)p - \frac{i\xi x_0}{2}\delta n b + \frac{1}{\sqrt{2}}(\varepsilon_l^* a_+ + \varepsilon_l a_-^\dagger), \\ \dot{b} &= (-i\omega_m - \gamma_m)b + \frac{i\xi x_0}{2}p - ig_d\sigma_-, \\ \dot{\sigma}_- &= (-i\omega_q - \gamma_q)\sigma_- + ig_db\sigma_z, \\ \dot{\sigma}_z &= -2\gamma_q(\sigma_z + 1) - 2ig_d(\sigma_+ b - b^\dagger \sigma_-), \end{aligned} \quad (\text{S2})$$

where  $\delta n = a_+^\dagger a_+ - a_-^\dagger a_-$  denotes the population inversion operator and  $\gamma_q$  is the decay rate of the TLS. The noise terms are negligible with a strong driven field. To obtain the mechanical gain, we set  $\partial p/\partial t = 0$ ,  $\partial \sigma_-/\partial t = 0$ , and  $\partial a_\pm/\partial t = 0$  with  $\gamma, \gamma_q \gg \gamma_m$ , which leads to the steady-state values

$$\begin{aligned} p &= \frac{1}{i(2J - \omega_m) + 2\gamma} \left[ -\frac{i\xi x_0}{2}\delta n b + \frac{1}{\sqrt{2}}(\varepsilon_l^* a_+ + \varepsilon_l a_-^\dagger) \right], \\ \sigma_- &= -\frac{g_d(\omega_q - \omega_m) + ig_d\gamma_q}{\gamma_q^2 + (\omega_q - \omega_m)^2 + 2g_d^2 n_b} b, \\ a_+ &= \frac{\varepsilon_l(2i\omega_- + 2\gamma + i\xi x_0 b)}{2\sqrt{2}\alpha - i4\sqrt{2}\gamma\Delta}, \\ a_- &= \frac{\varepsilon_l(2i\omega_+ + 2\gamma + i\xi x_0 b^\dagger)}{2\sqrt{2}\alpha - i4\sqrt{2}\gamma\Delta}, \end{aligned} \quad (\text{S3})$$

with  $n_b = b^\dagger b$  and

$$\omega_\pm = -\Delta \pm J, \quad \alpha = J^2 + \gamma^2 - \Delta^2 + \frac{\xi^2 x_0^2}{4} n_b.$$

Substituting Eq. (S3) into the dynamical equation of the mechanical mode results in

$$\dot{b} = (-i\omega_m + i\omega' + G - \gamma_m)b + C, \quad (\text{S4})$$

where

$$\begin{aligned} \omega' &= \frac{g_d^2(\omega_q - \omega_m)}{\gamma_q^2 + (\omega_q - \omega_m)^2 + 2g_d^2 n_b} - \frac{\xi^2 x_0^2(2J - \omega_m)}{16\gamma^2 + 4(2J - \omega_m)^2} - \frac{\xi^2 x_0^2 \Delta |\varepsilon_l|^2}{[2(2J - \omega_m)^2 + 8\gamma^2](\alpha^2 + 4\Delta^2 \gamma^2)}, \\ C &= \frac{i|\varepsilon_l|^2 \xi x_0}{2i(2J - \omega_m) + 4\gamma} \cdot \frac{(\gamma - iJ)\alpha + 2\Delta^2 \gamma}{\alpha^2 + 4\Delta^2 \gamma^2}, \end{aligned}$$

and the mechanical gain  $G = G_0 + G_d$  with

$$\begin{aligned} G_0 &= \frac{\xi^2 x_0^2 \gamma \delta n}{2(2J - \omega_m)^2 + 8\gamma^2} - \frac{\Delta \gamma (2J - \omega_m) \xi^2 x_0^2 |\varepsilon_l|^2}{[2(2J - \omega_m)^2 + 8\gamma^2](\alpha^2 + 4\Delta^2 \gamma^2)}, \\ G_d &= -\frac{g_d^2 \gamma_q}{\gamma_q^2 + (\omega_q - \omega_m)^2 + 2g_d^2 n_b}. \end{aligned} \quad (\text{S5})$$

Fig. S1(a) shows  $G$  as a function of  $\Delta$  with different inter-resonator tunneling rate  $J$ , indicating an optimized phonon lasing can be achieved by choosing  $J/\omega_m = 0.5$ .



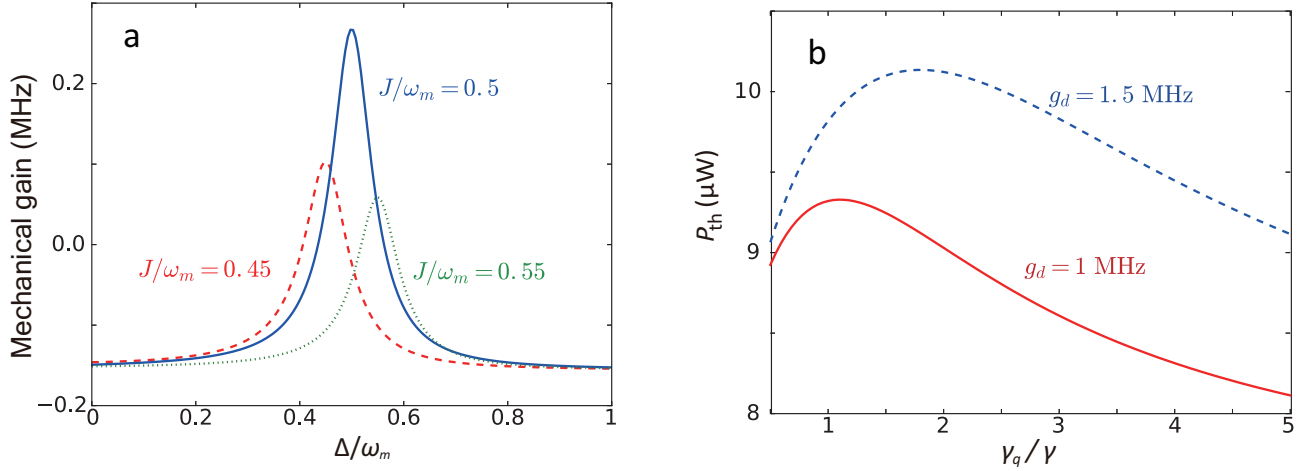
## II. THRESHOLD OF THE DEFECT-ASSISTED PHONON LASER

With the mechanical gain at hand, we can derive the threshold pump power  $P_{\text{th}}$  from the threshold condition  $G = \gamma_m$  [S1], i.e.

$$\begin{aligned} P_{\text{th}} &= P_{\text{th},0} + P_{\text{th},d} \\ P_{\text{th},0} &= \frac{2[(2J - \omega_m)^2 + 4\gamma^2](\omega_c + J)\gamma_m}{(\xi x_0)^2} + \frac{\hbar\Delta(2J - \omega_m)(\omega_c + J)|\varepsilon_l|^2}{\alpha^2 + 4\Delta^2\gamma^2}, \\ P_{\text{th},d} &= \frac{2g_d^2\gamma_q(\omega_c + J)[(2J - \omega_m)^2 + 4\gamma^2]}{\xi^2 x_0^2 [\gamma_q^2 + (\omega_m - \omega_q)^2 + 2g_d^2 n_b]}. \end{aligned} \quad (\text{S6})$$

The first term of  $P_{\text{th},0}$  is exactly the same as given in Ref. [S1] for  $\Delta = 0$ . Clearly, even for this resonant case, both the mechanical gain  $G$  and the threshold power  $P_{\text{th}}$  can be obviously different due to the defects.

We plot the threshold value  $P_{\text{th}}$  as a function of the TLS decay rate in Fig. S1(b). The threshold first increases but then decreases again, which is reminiscent of the loss-induced suppression and revival of an optical Raman laser as demonstrated recently in coupled passive resonators [S2].



**Figure S1:** (a) Mechanical gain in the defect-assisted phonon laser. (b) The threshold power of the defect-assisted phonon laser. The parameters are  $\gamma_q = \gamma$ ,  $g_d = 1$  MHz in (a) and  $J = 0.5\omega_m$ ,  $\Delta = 0.5\omega_m$  in (b). We choose  $\omega_q = \omega_m$  and  $P_l = 10 \mu\text{W}$  in both the two figures .

## III. TURNING POINTS OF THE MECHANICAL GAIN

Figure 2(b) of the main text shows the mechanical gain as a function of the TLS decay rate, in which the minimum value of mechanical gain does not correspond to the EP of the effective  $\mathcal{PT}$ -symmetric system composed of the active mechanical mode and the lossy TLS model. For  $\omega_q = \omega_m$ , we set  $\partial G/\partial \gamma_q = 0$  and obtain the turning point, i.e.

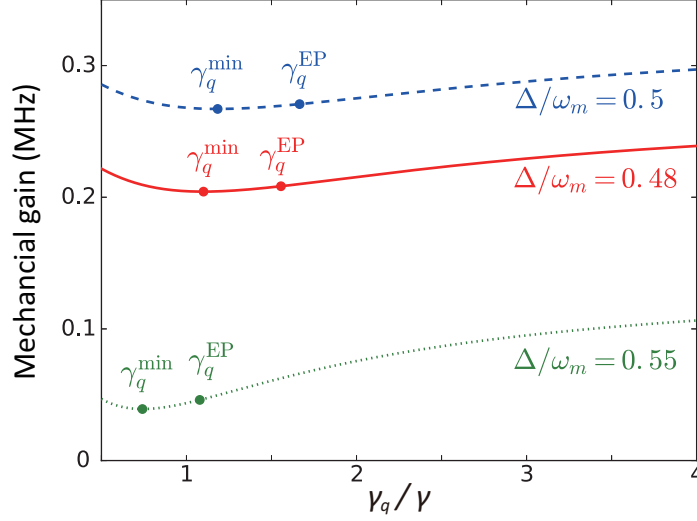
$$\gamma_q^{\text{min}} = \sqrt{2n_b g_d}, \quad (\text{S7})$$

while the EP emerges at

$$\gamma_q^{\text{EP}} = \gamma'_m + 2\sqrt{n_b g_d}, \quad (\text{S8})$$

where  $\gamma'_m = \gamma_m - G_0$ , and  $n_b$  is the stationary value of the phonon number. Figure S2 shows the turning points of the mechanical gain and EPs with different detuning. We can see that the turning points and EPs changes with different detuning. This slight shift of the turning point from the exact position of the EP was also observed in Ref. [S2]. If  $\gamma_q < \gamma_q^{\text{min}}$ , the system has less mechanical gain as the defect loss  $\gamma_q$  increases, which is due to the  $\mathcal{PT}$ -symmetric phase, where the active phonon mode partially or completely compensates the loss induced by the defect TLS. If

$\gamma_q$  exceeds  $\gamma_q^{\min}$  until reaches  $\gamma_q^{\text{EP}}$ , the supermode distributions are strongly affected by  $\gamma_q$ , which results in a slight growth of the mechanical gain. If  $\gamma_q$  reaches  $\gamma_q^{\text{EP}}$ , the system enters into the broken- $\mathcal{PT}$ -symmetric phase, leading to the obvious growth of the mechanical gain.



**Figure S2:** Mechanical gain in the defect-assisted phonon laser as a function of the TLS decay rate  $\gamma_q$ , for different detunings. The parameters are  $J = 0.5\omega_m$ ,  $\omega_q = \omega_m$  and  $P_l = 10 \mu\text{W}$ .

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